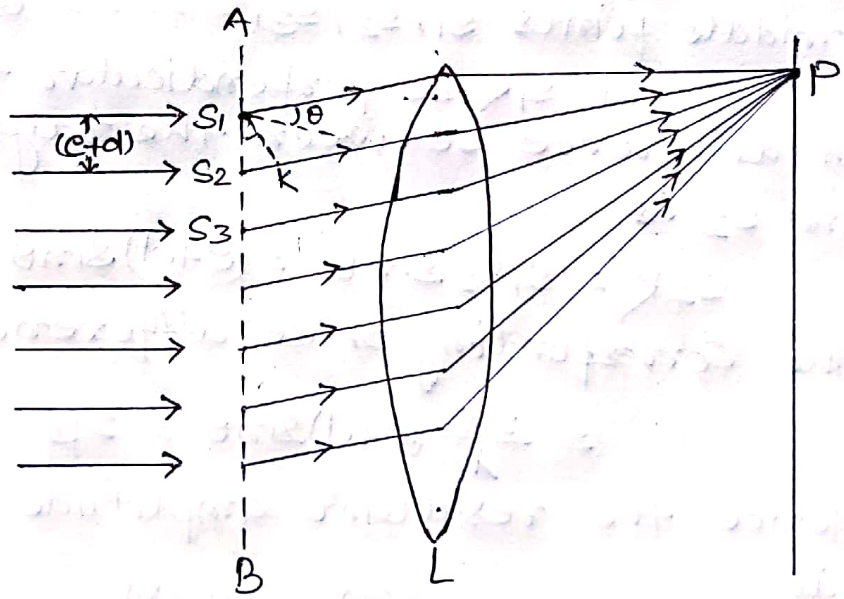
prof. vidya Sagar
Department of physics
Raja Singh College, Siwan

B.Sc (part-II) Hons
paper - iii

plane Transmission Diffraction Grating. condition for maxima and minima.

plane Transmission Diffraction Grating :- A diffraction grating is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one another by equal opaque spaces. It is made by ruling a large number of fine, equi-distance and parallel line on an optically-plane glass plate with a dimond ~~plate~~ point. The ruling scatter the light and are effectively opaque while the unruled parts transmit light and act as slits.



Let AB be the section of a plane transmission grating the length of the slits being perpendicular to the plane of the paper. Let 'e' be the width of each slit and 'd' the width of each opaque space between the slits. Then (e+d) is called 'grating element' The point in two Consecutive slits separated by the distance (e+d) are called the 'corresponding points'.

Let a parallel beam of monochromatic light of wave length λ be incident normally on the grating. By Huygen's principle all the points in each slit send out secondary wavelets in all directions. By the theory of Fraunhofer diffraction at a single slit, the wavelets from all points in a slit diffracted in a direction θ are equivalent to a single wave of amplitude $\frac{A \sin\alpha}{\alpha}$ starting from the middle point of the slit, where $\alpha = \frac{\pi}{\lambda} e \sin\theta$.

Thus if N be the total number of slits in the grating, the diffracted rays from all the slits are equivalent to N parallel rays, one each from the middle points $S_1, S_2, S_3, -----$ of the slit.

Let S_1K be perpendicular to S_2K. Then the path difference between the rays from the slits S_1 and S_2 is

$$S_2K = S_1S_2 \sin\theta = (e+d)\sin\theta$$

The corresponding phase difference

$$= \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$$

Hence the resultant amplitude in the direction θ is

$$R = \frac{A\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

The resultant intensity I is therefore given by

$$I = R^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin\beta} \quad \text{——— (1)}$$

The first factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ gives a diffraction pattern due to a single slit, while the second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the interference pattern due to N slits. Let us consider the intensity distribution due to the second factor.

<u>Principal Maxima:</u>— when $\sin\beta = 0$

$$\beta = \pm n\pi, \text{ where } n = 0,1,2,3, \text{-----}$$

We have $\sin N\beta = 0$ and thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$, i.e. indeterminate. Let us find its value by the usual method of differentiating the numerator and the denominator. Thus

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi} \frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N(\pm n\pi)}{\cos(\pm n\pi)} = \pm N$$

The intensity is the

$$I = \frac{A^2\sin^2\alpha}{\alpha^2} \cdot N^2$$

which is the maximum. These maxima are most intense and are called 'principal maxima'. They are obtained in the direction given by

$$\beta = \pm n\pi$$

or, $$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

or, $$(e+d)\sin\theta = \pm n\lambda \quad \text{———} \quad (2)$$

where $n = 0,1,2,3,$ ---- For $n=0$ we get the 'zero order maximum'. For $n = \pm 1, \pm 2, \pm 3,$ ----- we obtain the first, second, third ---- order principal maxima respectively. The $\pm$ sign show that there are two equal principal maxima for each order lying on either side of the zero-order maximum.

The positions of principal maxima given by equation (2) are the Same as given in a two slit diffraction pattern. Thus in a diffraction grating the position of principal maxima do not alter whether the Number of slits is 2 or $N (N>2)$, provided the grating element is the Same.

Minima:— When $\sin N\beta = 0$, but $\sin\beta \neq 0$ then

$$\frac{\sin N\beta}{\sin\beta} = 0$$

and hence the intensity $I = 0$

which is a minimum. These minima are obtained in the directions given by

$$\sin N\beta = 0$$

or, $$N\beta = \pm m\pi$$

or, $$N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

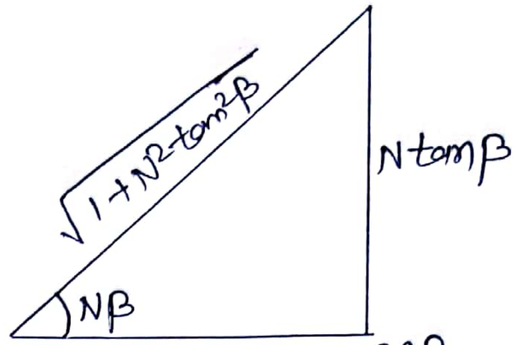
or, $$N(e+d)\sin\theta = \pm m\lambda \quad \text{(3)}$$

where m takes all integral values except $0, N, 2N$ ------ nN, because these values of m makes $\sin\beta = 0$. which gives principal maxima.

It is clear from above that $m=0$ gives a principal maximum $m = 1, 2, 3, \ldots (N-1)$ give minima ~~between two~~ and then $m = N$ gives again a principal maximum. Thus there are $(N-1)$ minima between two Consecutive principal maxima.

Secondary Maxima:— As there are $(N-1)$ minima between two consecutive principal maxima, there must be $(N-2)$ other maxima between two principal maxima. There are Called 'Secondary maxima'.

These position are obtained by differentiating (1) with respect to β and equating it to zero.



Thus

$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2}\, 2\left[\frac{\sin N\beta}{\sin\beta}\right] \frac{N\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta} = 0$$

or, $N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0$

or, $\tan N\beta = N\tan\beta$ ——— (4)

To find the value of $\frac{\sin^2 N\beta}{\sin^2\beta}$ under the condition (4), we make use of the triange. This gives

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{(1+N^2\tan^2\beta)\sin^2\beta} = \frac{N^2}{(1+N^2\tan^2\beta)\cos^2\beta}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

This shows that the intensity of the secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)\sin^2\beta}$, whereas the intensity of principal maxima is proportional to N^2. Therefore

$$\frac{\text{intensity of secondary maxima}}{\text{intensity of principal maxima}} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

Hence, greater the value of N, the weaker are secondary maxima. In an actual grating, N is very large. Hence these secondary maxima are not visible in the grating sepectrum.